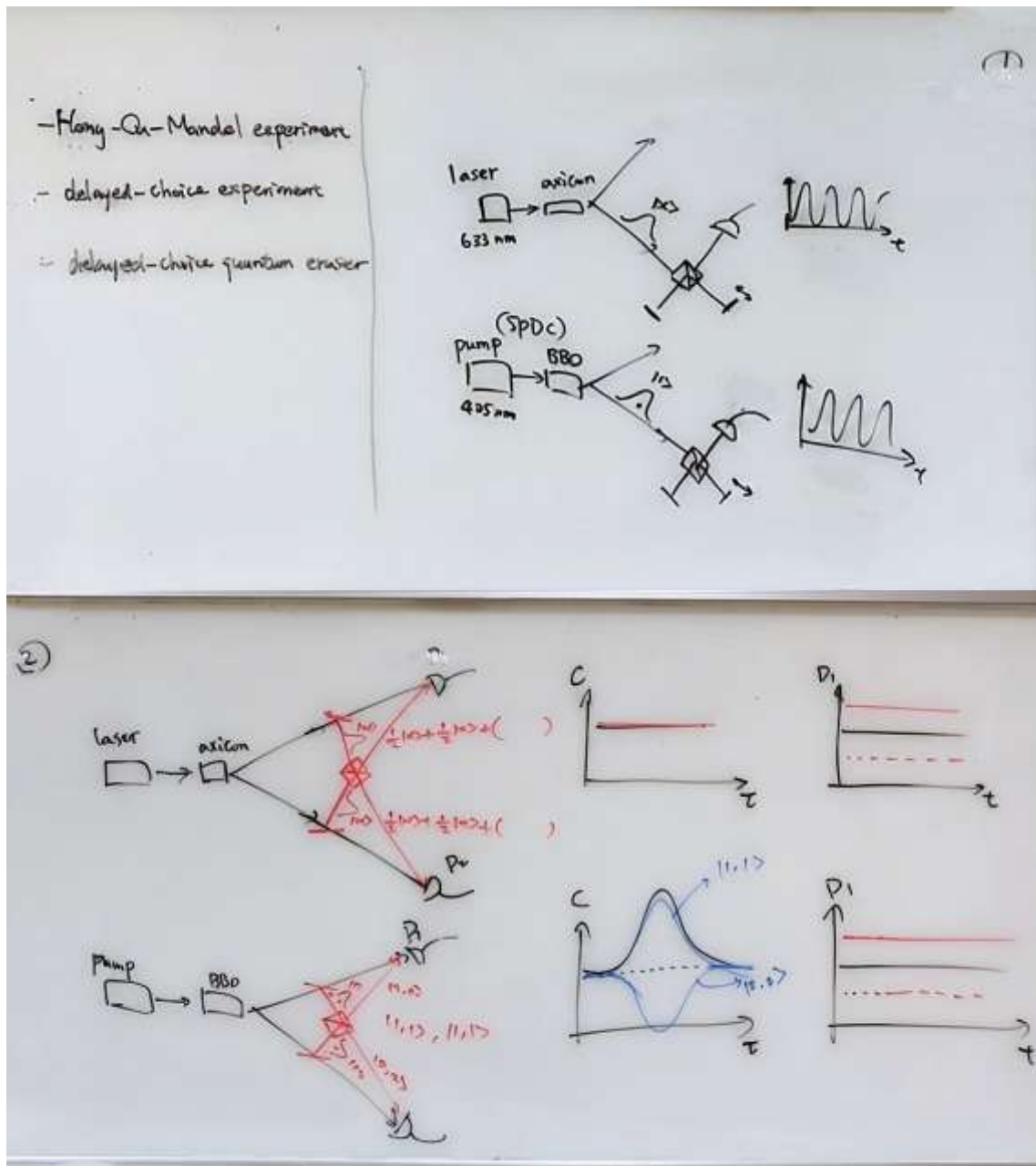




Lecture12 (optics) delayed-choice quantum eraser





Hong-Ou-Mandel

- Hong-Ou-Mandel experiment
- delayed-choice experiment
- delayed-choice quantum eraser

$[a_1, a_2] = 0$
 $[a_1, a_1^\dagger] = 1$
 $[a_2, a_2^\dagger] = 1$
 $[a_1, a_2^\dagger] = 0$
 $[a_1^\dagger, a_2] = 0$
 $[a_1^\dagger, a_2^\dagger] = 0$

$a_3 = \frac{1}{\sqrt{2}}(a_1 + i a_2)$
 $a_4 = \frac{1}{\sqrt{2}}(i a_1 + a_2)$

$a_1^\dagger = \frac{1}{\sqrt{2}}(a_3^\dagger - i a_4^\dagger)$
 $a_2^\dagger = \frac{1}{\sqrt{2}}(i a_3^\dagger + a_4^\dagger)$

④

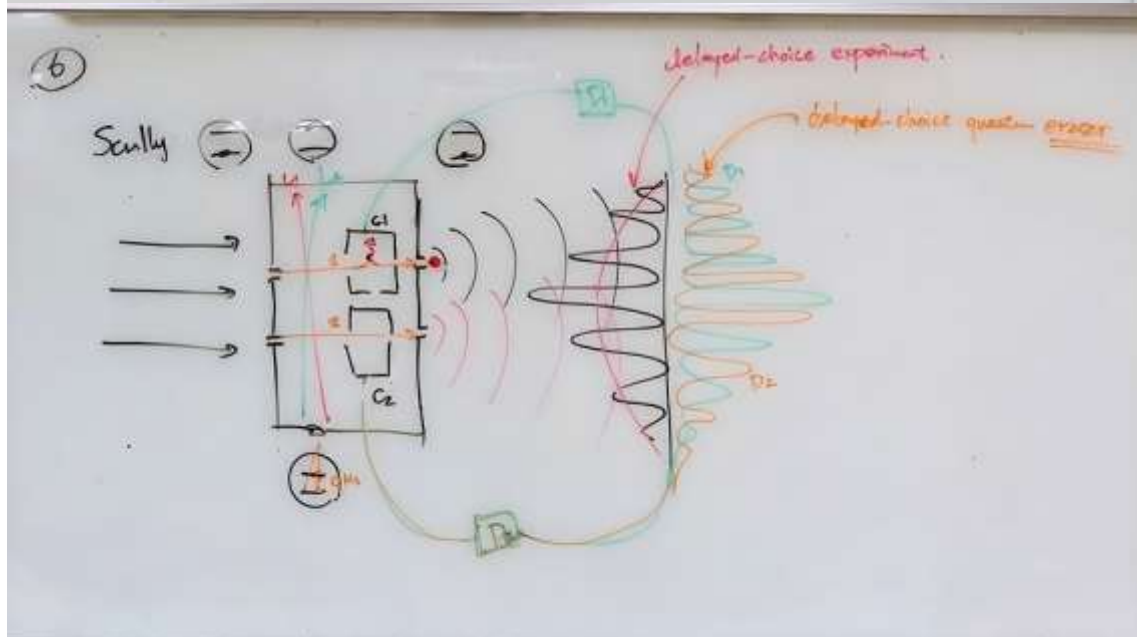
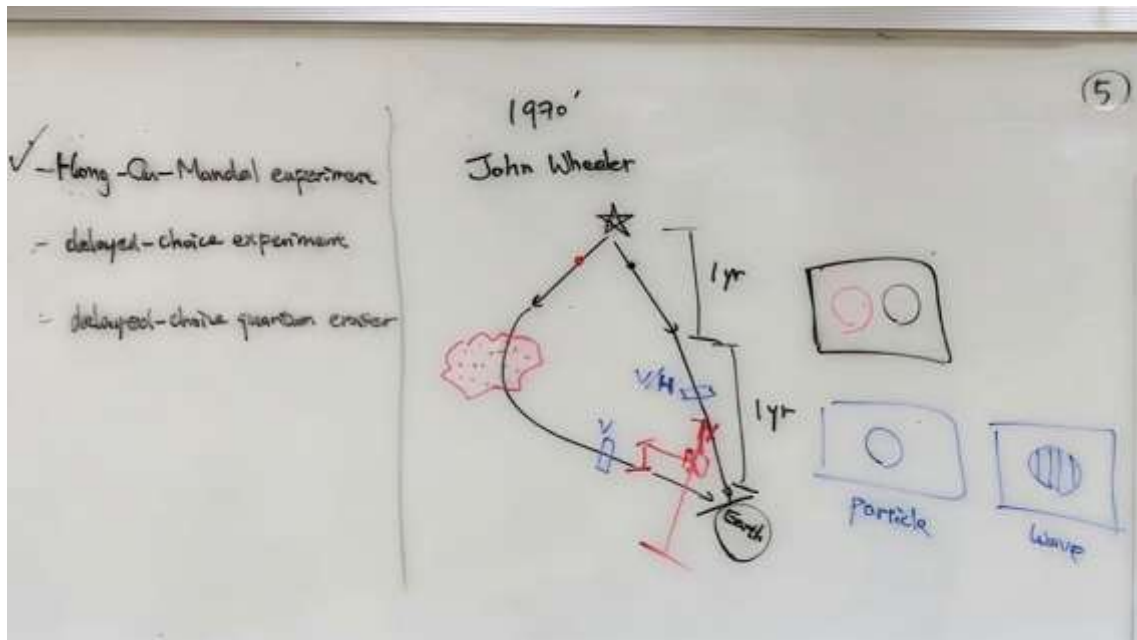
$|0, 0, 0, 0, \dots\rangle$

$$|1\rangle_0 |1\rangle_1 = a_0^\dagger a_1^\dagger |0\rangle$$

$$= \left(\frac{1}{\sqrt{2}}\right)^2 (a_0^\dagger + i a_3^\dagger)(i a_2^\dagger + a_4^\dagger) |0\rangle |0\rangle$$

$$= \left(\frac{1}{\sqrt{2}}\right)^2 (i a_0^\dagger a_2^\dagger + i a_3^\dagger a_4^\dagger + a_0^\dagger a_4^\dagger - a_3^\dagger a_2^\dagger) |0\rangle |0\rangle_2 |0\rangle_3$$

$$= \left(\frac{1}{\sqrt{2}}\right)^2 (i |2\rangle_0 |0\rangle_3 + i |0\rangle_0 |2\rangle_3 + |1\rangle_0 |1\rangle_3 - |1\rangle_0 |1\rangle_3) - |1\rangle_3 |1\rangle_2$$



✓ - Hong-Ou-Mandel experiment
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$|k_1\rangle = \cos(kx + wt)$
 $|k_2\rangle = \cos(kx - wt)$ (entangled state)

$$|\psi\rangle = |k_1\rangle|H\rangle + |k_2\rangle|V\rangle$$

$$P = \langle\psi|\psi\rangle = \langle k_1|k_1\rangle\langle H|H\rangle + \langle k_1|k_2\rangle\langle H|V\rangle + \langle k_2|k_1\rangle\langle V|H\rangle + \langle k_2|k_2\rangle\langle V|V\rangle$$

$\langle H|V\rangle = 0$

8)

$$\hat{\rho} = |\psi\rangle\langle\psi|$$

$$= (|k_1\rangle|H\rangle + |k_2\rangle|V\rangle)(\langle k_1|\langle H| + \langle k_2|\langle V|) \left(\frac{1}{\sqrt{2}}\right)^2$$

$$= \left(\frac{1}{\sqrt{2}}\right)^2 \left(|k_1\rangle\langle k_1| |H\rangle\langle H| + |k_2\rangle\langle k_2| |V\rangle\langle V| + |k_1\rangle\langle k_2| |H\rangle\langle V| + |k_2\rangle\langle k_1| |V\rangle\langle H| \right)$$

reduced density matrix

$$\hat{\rho}' = \text{Tr}_{\text{ph}}(\hat{\rho}) = \langle H|\hat{\rho}|H\rangle + \langle V|\hat{\rho}|V\rangle = \left(\frac{1}{\sqrt{2}}\right)^2 \left(|k_1\rangle\langle k_1| + |k_2\rangle\langle k_2| \right)$$

(partial trace)

$$\hat{\rho}' = \left(\frac{1}{\sqrt{2}}\right)^2 \begin{pmatrix} |k_1\rangle + |k_2\rangle & |k_1\rangle - |k_2\rangle \\ |k_1\rangle - |k_2\rangle & |k_1\rangle + |k_2\rangle \end{pmatrix}$$

Symmetric
 $|k_1\rangle = \frac{1}{\sqrt{2}}(|H\rangle + |V\rangle)$
 $|k_2\rangle = \frac{1}{\sqrt{2}}(|H\rangle - |V\rangle)$
 Anti-symmetric

$N^2(|a\rangle - |b\rangle) \frac{1}{\sqrt{2}} = \frac{1}{\sqrt{2}} \dots$
 $|H\rangle (|a\rangle + |b\rangle) \frac{1}{\sqrt{2}} = H \dots$





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projective measurement

$$\hat{P}_a |\psi\rangle = (\langle a | \psi \rangle) |a\rangle = |a\rangle \langle a | \psi \rangle$$

$$\hat{P}_a = |a\rangle \langle a|$$

projection operator ($\hat{P}_a^2 = \hat{P}_a$)

$$|\psi\rangle = (|h\rangle + |k\rangle) |a\rangle \left(\frac{1}{\sqrt{2}}\right) + (|h\rangle - |k\rangle) |b\rangle \left(\frac{1}{\sqrt{2}}\right)$$

$$\langle \psi | \psi \rangle = 1$$

$$|\psi_a\rangle = \hat{P}_a |\psi\rangle = \left(\frac{1}{\sqrt{2}}\right)^2 (|h\rangle + |k\rangle) |a\rangle$$

$$\langle \psi_a | \psi_a \rangle = \left(\frac{1}{\sqrt{2}}\right)^2 (1+1) = \frac{1}{2}$$

$$|\psi_a\rangle = \frac{\hat{P}_a |\psi\rangle}{\sqrt{\langle \psi_a | \psi_a \rangle}} = \frac{1}{2} (|h\rangle + |k\rangle) |a\rangle$$

$$\hat{P}_a = |\psi_a\rangle \langle \psi_a| = \frac{\hat{P}_a |\psi\rangle \langle \psi| \hat{P}_a}{\langle \psi_a | \psi_a \rangle} = \frac{1}{2} (|h\rangle + |k\rangle) \langle h| + \langle k|) |a\rangle \langle a|$$

$$\hat{P}_a = |\psi_a\rangle \langle \psi_a| = \frac{\langle \psi_a | \psi_a \rangle}{\langle \psi_a | \psi_a \rangle} = \frac{1}{2} (|h\rangle \langle h| + |h\rangle \langle k| + |k\rangle \langle h| + |k\rangle \langle k|)$$

$\hat{P}_a = T_{P_a}(\hat{P}) = \frac{1}{2} (|h\rangle \langle h| + |h\rangle \langle k| + |k\rangle \langle h| + |k\rangle \langle k|)$

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